Distributing Quantum Information with Microwave Resonators in Circuit QED

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Motivation

- Investigate new regimes of matter-light interaction in electronic circuits (Quantum optics, cavity quantum electrodynamics)

- Quantum circuits for information processing (Quantum computation)

- Interfaces between different physical systems (Quantum hybrids)
Outline

- Quantum Mechanics with Superconducting Circuits (microresonator in the quantum regime)

- Circuit QED (microresonator as noise filter + qubit readout + study of matter-light coupling)

- Generation of entangled 2-qubit and 3-qubit states (microresonator for quantum information distribution)

- Hybrid quantum computation with Rydberg atoms and superconducting circuits (microresonator as interface to other quantum objects)
Classical and Quantum Electronic Circuit Elements

basic circuit elements: charge on a capacitor:

\[
\frac{1}{\sqrt{2}} \left( \begin{array}{c}
\text{charge on a capacitor}
\end{array} + \begin{array}{c}
\text{charge on a capacitor}
\end{array} \right)
\]

current or magnetic flux in an inductor:

\[
\frac{1}{\sqrt{2}} \left( \begin{array}{c}
\text{current or magnetic flux in an inductor}
\end{array} + \begin{array}{c}
\text{current or magnetic flux in an inductor}
\end{array} \right)
\]

quantum superposition states:

• charge \( q \)
• flux \( \phi \)

[Review: M. H. Devoret, A. Wallraff and J. M. Martinis, condmat/0411172 (2004)]

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Constructing Linear Quantum Electronic Circuits

basic circuit elements:

\[ H = \frac{\phi^2}{2L} + \frac{q^2}{2C} \]

classical physics:

\[ \hat{H} = \frac{\hat{\phi}^2}{2L} + \frac{\hat{q}^2}{2C} \]

quantum mechanics:

\[ = \hbar \omega (\hat{a}^\dagger \hat{a} + \frac{1}{2}) \]

\[ [\hat{\phi}, \hat{q}] = i\hbar \]

harmonic LC oscillator: \( \omega = \frac{1}{\sqrt{LC}} \sim 5 \text{ GHz} \)

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1D Cavity with large Vacuum Field

optical microscope image of sample fabricated at FIRST (Nb on sapphire)

electric field across resonator in vacuum state \((n=0)\):

\[ E_{0, \text{rms}} \approx 0.2 \, \text{V/m} \quad \text{for} \quad \omega_r/2\pi \approx 6 \, \text{GHz} \]

\( \times 10^6 \) larger than \( E_0 \)
in 3D microwave cavity
Storing Photons and Controlling their Life Time

measuring the life time:

quality factor:

\[ Q = \frac{\nu_r}{\delta \nu_r} \approx 10^2 - 10^5 \]

photon lifetime:

\[ T_\kappa = \frac{1}{\kappa} \approx 10\,\text{ns} - 10\,\mu\text{s} \]


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Constructing Non-Linear Quantum Electronic Circuits

basic circuit elements:

- Josephson junction: a non-dissipative nonlinear element (inductor)

anharmonic oscillator:

\[ L_J(\phi) \]

non-linear energy level spectrum:

\[ L_J(\phi) = \left( \frac{\partial I}{\partial \phi} \right)^{-1} = \frac{\phi_0}{2\pi I_c} \frac{1}{\cos(2\pi \phi/\phi_0)} \]

electronic artificial atom

|e⟩

|g⟩

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Artificial atom: Cooper Pair Box Qubit

quantum state:
number $\hat{n}$ of Cooper pairs on island

transmon-design for increased charge noise resilience:

How to operate circuits quantum mechanically?

recipe:

- avoid dissipation
- work at low temperatures
- isolate quantum circuit from environment

$T \sim 0.01 \text{ K}$
Setup

resonator+ transmon chip:  Dilution fridge (20mk):

Sampleholder:  Box with B-field coils:
Superconducting Artificial Atoms and Molecules

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Cavity Quantum Electrodynamics

interaction of atom and photon in a cavity

Jaynes-Cummings Hamiltonian

\[ H = \hbar \omega_r \left( a^{\dagger}a + \frac{1}{2} \right) + \frac{\hbar \omega_a}{2} \sigma^z + \hbar g (a^{\dagger}\sigma^- + a\sigma^+) + H_\kappa + H_\gamma \]

strong coupling limit: \( g = \frac{dE_0}{\hbar} > \gamma, \kappa \)

D. Walls, G. Milburn, Quantum Optics (Springer-Verlag, Berlin, 1994)

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Our Circuit Realization of Cavity QED

Coherent quantum mechanics with individual photons and qubits
[S. Haroche & J. Raimond]

in superconducting circuits:

Circuit quantum electrodynamics

strong designable coupling


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Resonant coupling

qubit 1: transition frequency: \( \omega_{ge} \approx \sqrt{8E_C E_J} = \sqrt{8E_C E_{J,\text{max}}} \, |\cos(\pi \Phi / \Phi_0)| \)

resonator: • direct coupling (\( g \sim 130 \text{ MHz} \))
Resonant Vacuum Rabi Mode Splitting …

... with one photon \((n = 1)\):

very strong coupling:

\[
g_{ge}/\pi = 308 \text{ MHz} \\
\kappa, \gamma < 1 \text{ MHz} \\
g_{ge} \gg \kappa, \gamma
\]

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Tavis-Cummings model: Increase number of qubits

Coupling scales with number $N$ of atoms $d \propto \sqrt{N}$

$|1\rangle$ \hspace{1cm} $|\text{gg...g, 0}\rangle$ \hspace{1cm} $|0\rangle$ \hspace{1cm} $|0\rangle$ \hspace{1cm} $|0\rangle$

$|0\rangle$ \hspace{1cm} $|\text{gg...g, 0}\rangle$ \hspace{1cm} $|e\rangle$ \hspace{1cm} $|e\rangle$ \hspace{1cm} $|e\rangle$

$\nu_{r}$ \hspace{1cm} $\nu_{ge}$ \hspace{1cm} $\nu_{ge}$ \hspace{1cm} $\nu_{ge}$

$|N,1+\rangle$ \hspace{1cm} $|N,1-\rangle$

$\sqrt{N}g/\pi$

[J. M. Fink et al., *PRL* 103, (2009)]

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Rabi Splitting with $N = 1, 2, 3$ Qubits and 1 Photon

at degeneracy: two bright states, $N - 1$ dark states

[J. M. Fink et al., PRL 103, (2009)]
Dispersive regime

resonant:

dispersive (qubit detuned from resonance: \( \Delta = |\omega_{ge} - \omega_r| \gg g \))

[Blais et al., PRA 69 (2004)]
Circuit QED – read out of qubit state

low power transmission measurement to determine qubit state:

dispersive Hamiltonian:

\[ H = \hbar(\omega_r + \chi \sigma_z) a^\dagger a + \frac{\hbar}{2}(\omega_a + \chi) \sigma_z \]

state-dependent frequency shift
Circuit QED – read out of qubit state

low power transmission measurement to determine qubit state:

dispersive Hamiltonian:

\[ H = \hbar (\omega_r + \chi \sigma_z) a^\dagger a + \frac{\hbar}{2} (\omega_a + \chi) \sigma_z \]

state-dependent frequency shift
low power transmission measurement to determine qubit state:

dispersive Hamiltonian:

\[ H = \hbar (\omega_r + \chi \sigma_z) a^\dagger a + \frac{\hbar}{2} (\omega_a + \chi) \sigma_z \]

state-dependent frequency shift -> \( \sigma_z \) determined
Preparation of non-classical photon states using sideband transitions.
Sideband transitions in circuit QED

• Qubit & cavity off-resonant: \( \varphi = \vert m \rangle \langle m' \vert \gg \hat{a} \)

• Transitions can be driven using strong external fields

[Chiorescu et al. Nature (2004); Wallraff et al. PRL (2007); Blais et al. PRA (2007); Liu et al. PRB (2007); P. J. Leek et al., PRB(R) (2009)]
Operations using blue sideband

Sideband can be used for exchange of information between qubit and photon
Preparation of \( n \) photon Fock states with blue sideband transitions

\[
\sqrt{n} \Omega_1
\]

Sideband Rabi frequency scales with \( \sqrt{n} \)

Qubit -
High Q -
Low Q -

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Sideband Rabi Oscillations with Fock States $n=0$ to $4$

Result: Scaling of Rabi frequency

$$\Omega_n = \sqrt{n} \Omega_1$$

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[P. J. Leek et al., PRL 104, 100504 (2010)]
Entangling two distant qubits

resonator can also be used as a ‘quantum bus’ to create an **entangled** state (a quantum state, where the single qubits lose their individuality)
Entangling two qubits using sideband transitions

$$|\psi\rangle = (|ge\rangle + |eg\rangle)/\sqrt{2}$$
Entanglement of superconducting qubits

Experiment:

\[
\text{Re } \Psi_{\text{Bell}} \quad \text{Im } \Psi_{\text{Bell}}
\]

Theory:

\[
\Psi_{\text{Bell}} = (|g\rangle \otimes |g\rangle + |e\rangle \otimes |e\rangle) / \sqrt{2}
\]

Fidelity: \(\text{Tr} \left[ (\rho_{\text{exp}}^{1/2} \rho_{\text{th}}^{1/2} \rho_{\text{exp}}^{1/2})^{1/2} \right]\) 86%

[Leek et al., PRB 79, 180511R (2009); Filipp et al., PRL 102, 200402 (2009).] recent data: M. Baur (ETH Zurich)

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Qubit interactions mediated via virtual photons.
Resonant and dispersive coupling

qubit 1: transition frequency: \( \omega_{ge} \approx \sqrt{8E_C E_J} = \sqrt{8E_C E_{J,max}} |\cos(\pi \Phi / \Phi_0)| \)

qubit 2: constant frequency (5.5 GHz)

resonator:  
- direct coupling (\( g \sim 130 \) MHz)
- mediated J-coupling (\( J \sim 20 \) MHz)

[Major et al., Nature 449 (2007)]

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Avoided level crossing

- cavity mediated coupling leads to an avoided crossing

\[ \psi_s = \frac{g_e + e_g}{\sqrt{2}} \]

\[ \psi_a = \frac{g_e - e_g}{\sqrt{2}} \]

- two-photon transition becomes allowed at avoided crossing

- formation of a dark state
Formation of dark state – drive symmetry

dark state condition: \[ \langle gg | H_d | \psi_{\text{dark}} \rangle = 0 \]

anti-symmetric drive:
\[ H_d = \epsilon \left( \frac{g^{(1)}}{\Delta} \sigma_+^{(1)} - \frac{g^{(2)}}{\Delta} \sigma_+^{(2)} \right) + h.c. \]

symmetric drive:
\[ H_d = \epsilon \left( \frac{g^{(1)}}{\Delta} \sigma_+^{(1)} + \frac{g^{(2)}}{\Delta} \sigma_+^{(2)} \right) + h.c. \]
Anti-symmetric drive/symmetric dark state

[S. Filipp, PRA 83, 063827 (2011)]

\[
\psi_s = \frac{(ge + eg)}{\sqrt{2}}
\]

\[
\psi_a = \frac{(ge - eg)}{\sqrt{2}}
\]
J-coupling for Bell-state generation (SWAP gate)

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} (|gg\rangle + |ee\rangle) \]

fidelity: 74%
(here limited by T1 \(\sim 280\) ns)
3-qubit entanglement for quantum teleportation.
Quantum Teleportation

Qubit A: |ψ⟩ = 

Qubit B, C: |Bell⟩ = \frac{1}{\sqrt{2}} ( |0⟩|0⟩ + |1⟩|1⟩ )

Bell state measurement:

If Bell state 1:

If Bell state 2:

|ψ⟩ = e^{-iσ_x/2}|ψ⟩ = ...
Quantum processor platform with 3-Qubits

Main parameters

**Transmon qubits**
- Full individual coherent qubit control via local charge and flux lines
- Large coupling strength to resonator $g \sim 300 - 350$ MHz
- Coherences times: $T_1 \sim 0.8 - 1.2$ s, $T_2 \sim 0.4 - 0.7$ s.

**Resonator**
- $f_0 \sim 8.625$ GHz

Teleportation Circuit

Teleportation: transmission of quantum bit (qubit A) from Alice to Bob using a pair of entangled qubits (qubits B+C)

\[ |\psi_A\rangle \]

Preparation of Bell state

Measurement + classical communication

Bell Measurement

implemented three qubit tomography at step III

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State tomography of the entangled three qubit state

**Example:** State to be teleported on qubit A is \( |\Psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle + i|1\rangle) \)

\[
|\varphi\rangle = \frac{1}{2} \left\{ |0_A0_B\rangle \otimes |\Psi\rangle_C \\
+ |0_A1_B\rangle \otimes \sigma_x|\Psi\rangle_C \\
+ |1_A0_B\rangle \otimes \sigma_x|\Psi\rangle_C \\
+ |1_A1_B\rangle \otimes (-\sigma_z\sigma_x)|\Psi\rangle_C \right\}
\]

\[\rho = |\varphi\rangle \langle \varphi|\]

Simulating measurement of qubit A and B with projection onto \( |0_A0_B\rangle \)

\[
\rho_C = \langle 0_A0_B | \rho_{ABC} | 0_A0_B \rangle = |\Psi\rangle \langle \Psi|
\]

fidelity 88%
Hybrid Quantum Computation.
Hybrid Cavity QED with Atoms and Circuits

combine the best properties of two worlds

very strong dipole interactions ...
... in quantum engineered electronic circuits

long coherence times ...
... of Rydberg atoms

on-chip trapping & guiding possibilities
Other hybrid (circuit QED) approaches:

- Spin ensembles (NV centers)
  [Kubo et al., PRL 105, 140502 (2010); Schuster et al., PRL 205, 140501 (2010)]

- Atomic ensembles (BEC)
  [Verdu, PRL 103, 043603 (2009)]

- Charged particles (Ions)
  [Tian et al., PRL 92, 247902 (2004)]

- Electrons on Helium
  [Schuster et al., PRL 105, 040503 (2010)]

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Experimental Setup

Photoexcitation region
Field ionization region

Superconducting chip

Pulse-tube cooler

Pulsed valve
Skimmer

4 K 50 K
MCP

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Driving Transitions

Energy level diagram showing the transitions between different energy levels (n,l) such as 33p, 32, 33s, 3s, 2s, and 1s. The diagram illustrates the ionization limit and the effect of a laser on the electron signal.
Summary

○ Photon storage in high-Q mode and entanglement generation using sideband transitions

○ Collective effects of multi-qubits coupled to a single resonator mode

○ Generation of 3-qubit entangled states for quantum teleportation

○ Hybrid system of Rydberg atoms and superconducting circuits for future quantum computation
Thanks for your attention.